Analytical and Numerical Methods in Differential Equations (Yanenko 100 and ANMDE 2021)

Conference Program and Book of Abstracts

23 – 27 August 2021

Virtual Host:

Suranaree University of Technology
Nakhon Ratchasima, Thailand

Day 1 : Monday, 23 August 2021

	Chair: Chaiyasena A. P.
13.00 – 13.05	Opening Address by **Assoc.Prof. Dr. Anan Tongraar** Acting Rector Suranaree University of Technology
13.05 – 13.10	Opening Remarks by **Prof. Dr. Eugene A. Bondar** Deputy Director Khristianovich Institute of Theoretical and Applied Mechanics
13.10 – 13.15	Opening Remarks by **Prof. Dr. Sibusiso Moyo** Deputy Vice Chancellor for Research, Innovation and Engagement Durban University of Technology
13.15 – 13.45	**Fomin V. M.** N.N. Yanenko — Siberian, Soldier and Scientist
13.45 – 14.15	**Il'in, V. P.** (postponed to Friday) The strategies and tactics of an intelligent mathematical modeling
14.15 – 14.40	**Peradzynski Z., Baghaturia G.** Double waves and Yanenko equation
Coffee Break	
	Chair: Pukhnachev V. P.
14.50 – 15.20	**Oberlack M., Hoyas S., Kraheberger S., Alcántara-Álvila F., Laux J., Klingenberg D., Hollmann P., Vallikivi M., Hultmark M., Bellani G., Talamelli A., Zimmerman S., Klewicki J.** Classical and statistical symmetries of turbulence – the basis of turbulent scaling laws of wall-bounded shear flows for arbitrary moments
15.20 – 15.45	**Wacławczyk M, Grebenev V. N., Oberlack M.** Conformal invariance of the 1-point statistics of the zero-isolines of $2d$ scalar fields in inverse turbulent cascades
15.45 – 16.10	**Andreev V. K., Lemeshkova E. N.** On the asymptotic behavior of inverse problems for parabolic equation
16.10 – 16.35	**Grebenev V. N., Demenkov A. G., Chernykh G. G.** Local equilibrium approximation in free turbulent flows: verification through the method of differential constraints
16.35 – 17.00	**Algazin S. D., Selivanov I. A.** (cancelled) About the flutter of an orthotropic plate rectangular in plan
Lunch / Dinner Break	
	Chair: Kaptsov O. V.
18.00 – 18.25	**Manganaro N.** Method of differential constraints for nonlinear wave problems
18.25 – 18.50	**Habibullin I. T.** Generalized invariant manifolds and their applications
18.50 – 19.15	**Dryuma V. S.** The Riemann spaces related to the Navier-Stokes equations
19.15 – 19.40	**Pavlov M. V.** Egorov hydrodynamic type systems
19.40 – 20.10	**Sergyeyev A.** Integrable systems in four independent variables from contact geometry

The Riemann Spaces Related to the Navier-Stokes Equations

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For solving system of the Navier-Stokes equations $\frac{\partial}{\partial t}\vec{V}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}=\mu\Delta\vec{V}+\vec{\nabla}P(\vec{x},t),\ \vec{\nabla}\cdot\vec{V}=0,$ where $\vec{V}(\vec{x},t)$ -is the fluid velocity, $P(\vec{x},t)$- is the pressure and μ- is the viscosity of liquid, was used their representation in form of the laws of conservations:

$$\frac{\partial}{\partial y}H(\vec{x},t)-\frac{\partial}{\partial x}E(\vec{x},t)=0,\frac{\partial}{\partial z}H(\vec{x},t)-\frac{\partial}{\partial x}B(\vec{x},t)=0,\frac{\partial}{\partial z}E(\vec{x},t)-\frac{\partial}{\partial y}B(\vec{x},t)=0$$

This allow us to introduce six-dimension Riemann space, equipped by the metric $ds^2=-2\,B(\vec{x},t)dt\,dv+2\,E(\vec{x},t)dt\,dw+2\,H(\vec{x},t)dv\,dw$ -2$(\int\frac{\partial}{\partial y}H(\vec{x},t)dz)dw^2+dt\,dx+dv\,dy+dw\,dz$ and to use it for construct an examples of solutions of the NS-equations. In particular case when the components of metric are of the form $B(\vec{x},t)=\frac{\partial^2}{\partial z^2}Q(\vec{x},t)$,$H(\vec{x},t)=\frac{\partial^2}{\partial x\partial z}Q(\vec{x},t)E(\vec{x},t)=\frac{\partial^2}{\partial y\partial z}Q(\vec{x},t)$ solutions of the NS-equations are expressed throw the function $Q(\vec{x},t)=\frac{\partial P}{\partial y}$, that is solution of the Monge-Ampere equation (MA):

$$2\,Q_{xyzz}^2-2\,Q_{yyxz}Q_{xzzz}-2\,Q_{xxyz}Q_{yzzz}+Q_{xxzz}Q_{zzyy}+Q_{xxyy}Q_{zzzz}=0. \tag{1}$$

Theorem 1. *The equation (1) determines function pressure $P(\vec{x},t)=\int(Q(\vec{x},t)dt)$ of flow and in particular cases admit reduction to the second order ODE of the form $y''+a_1(x,y)y'^3+3a_2(x,y)y'^2+3a_3(x,y)y'+a_4(x,y)=0$, where $a_i=a_i(x,y)$. This type of ODE's meet in theory of nonlinear dynamical systems $\dot{\vec{x}}=F_i(\vec{x})$ with polynomial right parts which have as the limit cycles and also strange attractors in their space of states.* As an example, we give the equation

$$\frac{d^2}{dx^2}y(x)-3\,\frac{\left(\frac{d}{dx}y(x)\right)^2}{y(x)}-2\,\frac{(k-6\,x+3)\left(\frac{d}{dx}y(x)\right)y(x)}{k+3}-12\,\frac{(y(x))^3x^2}{k+2}+\frac{(4\,k+12)\,(y(x))^3x}{k+2}+$$
$$+\frac{(-k-2)\,(y(x))^3}{k+2}-6\,\frac{(y(x))^4x^4}{(k+3)\,(k+2)}-2\,\frac{(-6-2\,k)\,(y(x))^4x^3}{(k+3)\,(k+2)}-2\,\frac{\left(k^2+3\,k+3\right)(y(x))^4x^2}{(k+3)\,(k+2)}=0,$$

which has a form similar to the equation $y''-3\,\frac{y'^2}{y}+\left(\alpha\,y-x^{-1}\right)y'+\epsilon\,xy^4+\frac{\delta}{x}y^2-\gamma\,y^3-\beta\,x^3y^4-\beta\,x^2y^3=0$, which is equivalent to the Lorenz-system $\dot{y}=rx-y-xz,\ \dot{z}=xy-bz,\ \dot{x}=\sigma(y-x),$ where: $\alpha\sigma=1,\ \beta\sigma^2=1,\ \gamma\sigma^2=b(\sigma+1),\ \delta\sigma=(\sigma+1),\epsilon\sigma^2=b(r-1)$.

A more general approach to study properties of the NS-equations is connected by using the 14-dimension space with local coordinates $x,y,z,t,u,v,w,p,\xi,\eta,\chi,\rho,q,\delta$.

Theorem 2. The metric: $ds^2=2\,dxdu+2\,dydv+2\,dzdw+(-W(\vec{x},t)w-V(\vec{x},t)v-U(\vec{x},t)u)\,dt^2+\left(-U(\vec{x},t)p-u\,(U(\vec{x},t))^2-uP(\vec{x},t)+w\mu\,\frac{\partial}{\partial z}U(\vec{x},t)-wU(\vec{x},t)W(\vec{x},t)\right)d\eta^2+$
$+\left(v\mu\,\frac{\partial}{\partial y}U(\vec{x},t)-vU(\vec{x},t)V(\vec{x},t)+u\mu\,\frac{\partial}{\partial x}U(\vec{x},t)\right)d\eta^2+2\,\eta d\xi+2\,d\rho d\chi+2\,dmdn+$
$+\left(-V(\vec{x},t)p-vP(\vec{x},t)-v\,(\vec{x},t))^2-V(\vec{x},t)W(\vec{x},t)w+v\mu\,\frac{\partial}{\partial y}V(\vec{x},t)-uU(\vec{x},t)V(\vec{x},t)\right)d\rho^2+$
$+\left(u\mu\,\frac{\partial}{\partial x}V(\vec{x},t)\right)d\rho^2+\left(-uU(\vec{x},t)W(\vec{x},t)-w\,(W(\vec{x},t))^2-wP(\vec{x},t)+w\mu\,\frac{\partial}{\partial z}W(\vec{x},t)\right)dm^2+$
$+\left(v\mu\,\frac{\partial}{\partial y}W(\vec{x},t)-vV(\vec{x},t)W(\vec{x},t)+u\mu\,\frac{\partial}{\partial x}W(\vec{x},t)-W(\vec{x},t)p\right)dm^2$, is the Ricci-flat on solutions of the NS-equations.

This metric belongs to the class of partially-projective spaces and their Cartan-invariants, $K=R(^i\ a\ j\ b)R(^j\ c\ i\ d)A^aA^bA^cA^d$, constructed on the basis of the Riemann curvature of the space are used then in theory of the NS-equations.

REFERENCES

1. Dryuma V. S., *The Riemann and Einstein geometries in the theory of ODE's, their appications and all that*, New Trends in Integrability and Partial Sovaability., **Kluwer academic Publisher**(A.B. Shabat at al.(eds.), 115–156 (2004).
2. Dryuma V. S., *On spaces related to the Navier-Stokes equations*, Buletinul Academiei de Stiinte a Republicii Moldova., **Matematica, 3(64)**, 107–110 (2010).