THE RIEMANN SPACES RELATED WITH NAVIER-STOKES EQUATIONS

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For solving Navier-Stokes equations

$$\frac{\partial}{\partial t}\vec{V} + \left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} = \mu\Delta\vec{V} + \vec{\nabla}P(\vec{x},t), \quad \vec{\nabla}\cdot\vec{V} = 0, \tag{1}$$

where $\vec{V}(\vec{x},t)$ is the fluid velocity, $P(\vec{x},t)$ is the pressure and μ is the viscosity of liquid, are used the conditions of their compatibility in the form of conservation laws $\frac{\partial}{\partial y}H(\vec{x},t) - \frac{\partial}{\partial x}E(\vec{x},t) = 0$, $\frac{\partial}{\partial z}H(\vec{x},t) - \frac{\partial}{\partial x}B(\vec{x},t) = 0$, $\frac{\partial}{\partial z}E(\vec{x},t) - \frac{\partial}{\partial y}B(\vec{x},t) = 0$.

The $6D$-space with the metric

$$ds^2 = -2\,B(t,x,y,z)dt\,dv + 2\,E(t,x,y,z)dt\,dw + 2\,H(t,x,y,z)dv\,dw -$$
$$-2\int\frac{\partial}{\partial y}H(t,x,y,z)dz\,dw^2 + dt\,dx + dv\,dy + dw\,dz. \tag{2}$$

is applied to construct an example of solutions of the NS-equations.

In particular case $B(\vec{x},t) = \frac{\partial^2}{\partial z^2}Q(t,x,y,z)$,$H(t,x,y,z) = \frac{\partial^2}{\partial x\partial z}Q(t,x,y,z)$, $E(t,x,y,z) = \frac{\partial^2}{\partial y\partial z}Q(t,x,y,z)$, the metric (2) is simplified and solutions of the NS-equations are expressed in terms of the function $P(x)$ that satisfies the Monge-Ampere equation.

A more general approach to obtain the solutions of the NS-equations is connected by using the $14D$ space with condition on the Ricci curvature $R_{ik} = 0$ on solutions of the NS system.

Theorem. *The metrics*

$$ds^2 = 2\,dxdu + 2\,dydv + 2\,dzdw + (-W(\vec{x},t)w - V(\vec{x},t)v - U(\vec{x},t)u)\,dt^2 + 2\,dtdp +$$
$$\left(-U(\vec{x},t)p - u\,(U(\vec{x},t))^2 - uP(\vec{x},t) + w\mu\,\frac{\partial}{\partial z}U(\vec{x},t) - wU(\vec{x},t)W(\vec{x},t)\right)d\eta^2 +$$
$$\left(v\mu\,\frac{\partial}{\partial y}U(\vec{x},t) - vU(\vec{x},t)V(\vec{x},t) + u\mu\,\frac{\partial}{\partial x}U(\vec{x},t)\right)d\eta^2 + 2\,\eta d\xi +$$
$$\left(-V(\vec{x},t)p - vP(\vec{x},t) - v\,(\vec{x},t))^2 - V(\vec{x},t)W(\vec{x},t)w + v\mu\,\frac{\partial}{\partial y}V(\vec{x},t)\right)d\rho^2 +$$

$$\left(u\mu\,\frac{\partial}{\partial x}V(\vec{x},t)-uU(\vec{x},t)V(\vec{x},t)\right)d\rho^2+2\,d\rho d\chi+$$

$$\left(-uU(\vec{x},t)W(\vec{x},t)-w\left(W(\vec{x},t)\right)^2-wP(\vec{x},t)+w\mu\,\frac{\partial}{\partial z}W(\vec{x},t)\right)dm^2+$$

$$\left(v\mu\,\frac{\partial}{\partial y}W(\vec{x},t)-vV(\vec{x},t)W(\vec{x},t)+u\mu\,\frac{\partial}{\partial x}W(\vec{x},t)-W(\vec{x},t)p\right)dm^2+2\,dmdn$$

is the Ricci-flat on solutions of the equations (1).

The Cartan Invariants $K=R(a,j,b,^i)R(c,i,d,^j)A^aA^bA^cA^d$ of the metrics are used to construct solutions of the equations NS and their properties are discussed.