

Invariants of E. Cartan and their applications to the theory of differential equations

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In the presented report, the theory of E. Cartan's invariants of multidimensional Riemann metrics related to non-linear differential equations, which have important applications in various areas of modern mathematical physics, will be considered in order to construct their exact solutions. As examples, we consider a 6-dimensional space with the metric

$${}^6ds^2 = 4U_x p dx dt + 4U_y p dy dt + (-2pUU_x - 2pU_{xxx} - 2\mu q U_y + 2UU_x) dt^2 + \\ + 2dx dp + 2dy dq + dU dt,$$

that is Ricci-flat on solutions of the well-known Kadomtsev-Petviashvili equation

$$(U_x)^2 + UU_{xx} + U_{xxx} + U_{xt} + \mu U_{yy} = 0$$

and the 14-dimensional Ricci-flat metric on solutions of the Navier-Stokes system of equations, describing the motion of an incompressible viscous fluid [1].

For build new examples of solutions of indicated differential equations the Invariants of E.Cartan [2]

$$S = R({}^i_{ajb})R({}^j_{cid})k^ak^bk^ck^d, \quad T = R({}^i_{ajb;cd})R({}^j_{eif;gh})k^ak^bk^ck^dk^ek^fk^gk^h.$$

and $Q = R({}_{ab})R({}_{cd})k^ak^bk^ck^d$, where $R({}^j_{cid})$ and $R({}_{cd})$ are the tensor of Riemann and Ricci-tensor of metric, k^q -components of vector field are applied and their properties are discussed.

Bibliography

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