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On Geodesic Equations of Riemannian Metrics Related to the Navier-Stokes Equations

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1.The $14D$ Riemann -metric in local coordinates $\vec{x} = [x, y, z, t, \eta, \rho, m, u, v, w, p, \xi, \chi, n]$

$$ds^2 = 2dx\,du + 2dy\,dv + 2dz\,dw + (-Uu - Vv - Ww)\,dt^2 + 2dt\,dp+$$
$$+Ad\eta^2 + 2d\eta\,d\xi + Bd\rho^2 + 2d\rho\,d\,\chi + Cdm^2 + 2dm\,dn,$$

where

$$A = (-UW + \mu\,U_x)\,w + (-UV + \mu\,U_y)\,v + \left(-U^2 - P + \mu\,U_x\right)u - Up,$$
$$B = (\mu\,V_z - VW)\,w + \left(-V^2 - P + \mu\,V_y\right)v + (-UV + \mu\,V_x)\,u - Vp,$$
$$C = \left(\mu\,W_z - \left(W^2 - P\right)\right)w + (\mu\,W_y - VW)\,v, (-UW + \mu\,W_x)\,u - Wp,$$

(U, V, W), P,- the components of velocity and pressure of liquid, has the Ricci-tensor of the form $R_{44} = U_x + V_y + W_z = 0$, $R_{55} = 0$, $R_{66} = 0$, $R_{77} = 0$ on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{V} + \left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} - \mu\Delta\vec{V} + \vec{\nabla}P = 0, \quad \vec{\nabla}\cdot\vec{V} = 0.$$

It belongs to the class of Riemann extensions of affinely-connected spaces, are equipped by partially-projective metrics due the conditions to part of coordinates

$$\ddot{\eta} = 0, \quad \ddot{\rho} = 0, \quad \ddot{m}, \quad \ddot{\xi} = 0, \quad \ddot{\chi} = 0, \quad \ddot{n} = o.$$

and the remaining geodesic equations have the form $\ddot{x}^k + \Gamma_{i^k_j}\dot{x}^i\dot{x}^j = 0$, $\ddot{\Psi}_k + T^i_k\Psi_i = 0$, where $x = (x, y, z, t)$, $\Psi_k = (u, v, w, p)$ and T^i_k do not depend on variables x.

2.The $6D$ Riemann -metric in local coordinates $\vec{x} = [x, y, z, t, v, w]$ looks as

$$ds^2 = dx\,dt + dy\,dv + dz\,dw - 2\,B(x, y, z, t)dt\,dv + 2\,E(x, y, z, t)dt\,dw+$$
$$+2\left(\frac{\partial^2}{\partial x\partial z}Q(x, y, z, t)\right)dv\,dw - 2\left(\frac{\partial^2}{\partial x\partial y}Q(x, y, z, t)\right)dw^2 \qquad (2).$$

Its tensor Ricci R_{ik} generally has 15 non-zeo components $R_{ik} \neq 0$ and the behavior of geodesic of such metric can be both regular or chaotic.

As example we find

$$t(s) = \frac{e^{C_1\,s}C_2}{C_1}, \quad v(s) = \frac{e^{C_1\,s}C_2 + C_2\,sC_1 + C_3\,C_1}{C_1},$$
$$w(s) = \frac{e^{C_1\,s}C_2 + C_2\,sC_1 + C_3\,C_1 + \ln(e^{C_1\,s}C_2)C_1 + {C_1}^2}{C_1},$$
$$\frac{d^2}{ds^2}x(s) + \left(\frac{d}{ds}x(s)\right)C_1 + \left(\frac{d}{ds}y(s)\right)C_1 + \left(\frac{d}{ds}z(s)\right)C_1 = 0,$$

and corresponding equations to the $\frac{d^2}{ds^2}y(s)$, $\frac{d^2}{ds^2}z(s)$...

With the help of solutions of geodesic equations, the fluid velocity and pressure components can be obtained by passing from the Navier-Stokes equations in Lagrangian variables to Euler variables.

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