

Multidimensional geometry of Riemann in theory of integration of the Navier-Stokes equations

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ABSTRACT

To integrate a system of Navier-Stokes equations, we consider an associated 14-dimensional space equipped with a Riemann metric, in which the Ricci tensor vanishes on the solutions of the system of equations. The metric belongs to the class of partially projective Riemannian spaces with scalar invariants equal to zero and is found in the theory of gravitational waves. E. Cartan invariants or classical Beltrami-Laplace invariants can be used to study metrics of this type.

The geodesic lines of the introduced metric are solutions to a system of four second-order nonlinear ODEs in coordinates (x, y, z, t) and four second-order linear ODEs for dual coordinates (u, v, w, p) with coefficients depending on the components of the curvature tensor of the Riemann space. The six additive coordinates are flat and they form a configuration of six straight lines as geodesics.

An important role in the theory of integration of the Navier–Stokes equations belongs to the conditions of their compatibility. They allow a geometric description based on the study of the properties of a six-dimensional space equipped with a Riemannian metric with special conditions for its components.

The coordinates (x, y, z, t) and (u, v, w, p) are dual to each other, and their properties are determined by the geometry of spaces of normal projective connectivity by E. Cartan for the corresponding pair of coordinates. They form the basis of a geometric approach to integrating the Navier-Stokes equations into Euler or Lagrange variables by studying corresponding metrics of four-dimensional or eight-dimensional Riemannian spaces.

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