MULTIDIMENSION RIEMANN METRICS TO INTEGRATING OF THE NAVIER-STOKES EQUATIONS

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Theorem 1. *The metric of the form*

$$ds^2 = 2\,dx^2 + 2\,dxdy + 2\,dxdu + 2\,dy^2 + 2\,dydz + 2\,dydv + 2\,dz^2 +$$

$$+2\,dzdw + 2\,dtdp + 2\,d\eta d\xi + 2\,d\rho d\chi + 2\,dmdn +$$

$$+Adt^2 + B d\eta^2 + C d\rho^2 + E dm^2, \tag{1}$$

where

$$A = 2 - U\,(x, y, z, t)\, u - V\,(x, y, z, t)\, v - W\,(x, y, z, t)\, w,$$

$$B = \left(-UW + \mu\,\frac{\partial}{\partial z}U\right) w + \left(-UV + \mu\,\frac{\partial}{\partial y}U\right) v + \left(\mu\,\frac{\partial}{\partial x}U - (U)^2 - P\right) u - Up,$$

$$C = \left(-VW + \mu\,\frac{\partial}{\partial z}V\right) w + \left(\mu\,\frac{\partial}{\partial y}V - (V)^2 - P\right) v + \left(-UV + \mu\,\frac{\partial}{\partial x}V\right) u - Vp,$$

$$E = \left(-\mu\,\frac{\partial}{\partial x}U - \mu\,\frac{\partial}{\partial y}V - (W)^2 - P\right) w + \left(-VW + \mu\,\frac{\partial}{\partial y}W\right) v +$$

$$+ \left(-UW + \mu\,\frac{\partial}{\partial x}W\right) u - Wp$$

is the Ricci-flat metric on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{U} + \left(\vec{U}\cdot\vec{\nabla}\right)\vec{U} - \mu\Delta\vec{U} + \vec{\nabla}P = 0, \qquad \vec{\nabla}\cdot\vec{U} = 0, \tag{2}$$

where $\vec{U} = [U(\vec{x},t), V(\vec{x},t), W(\vec{x},t)]$ *is the fluid velocity,* $P = P(\vec{x},t)$ *is the pressure,* μ *is the viscosity and* $\vec{x} = (x, y, z)$.

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Proposition. *The metrics (1) and*

$$y^2{d_x}^2+2\left(l\left(x,z\right)y^2+m(x,z)\right)d_xd_z+2\,d_yd_z+$$
$$\left(\left(l(x,z)\right)^2y^2-2\left(\frac{\partial}{\partial x}l(x,z)\right)y+2\,l\left(x,z\right)m(x,z)+2\,l\left(x,z\right)\right){d_z}^2 \tag{3}$$

that are applied to integration of the KdV-equation:

$$\frac{\partial}{\partial z}l\left(x,z\right)-3\,l\left(x,z\right)\frac{\partial}{\partial x}l\left(x,z\right)+\frac{\partial^3}{\partial x^3}l\left(x,z\right)=0,$$

the KP-equation:

$$\frac{\partial^2}{\partial x\partial t}n\left(x,y,t\right)-3\left(\frac{\partial}{\partial x}n\left(x,y,t\right)\right)^2-n\left(x,y,t\right)\frac{\partial^2}{\partial x^2}n\left(x,y,t\right)+$$
$$\frac{\partial^4}{\partial x^4}n\left(x,y,t\right)-\alpha^2\frac{\partial^2}{\partial y^2}n\left(x,y,t\right)=0,$$

the Schrödinger equation:

$$i\frac{\partial}{\partial t}\phi\left(x,y,t\right)-\kappa\frac{\partial^2}{\partial x^2}\phi\left(x,y,t\right)-K(x,y,t)\phi\left(x,y,t\right)=0,$$

may be used to construct an example of exact solutions of the system (2).

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