ISND-2026 · Yaroslavl · 21–25 September 2026 · e-poster

Riemann spaces associated with the Navier–Stokes equations: a geometric approach

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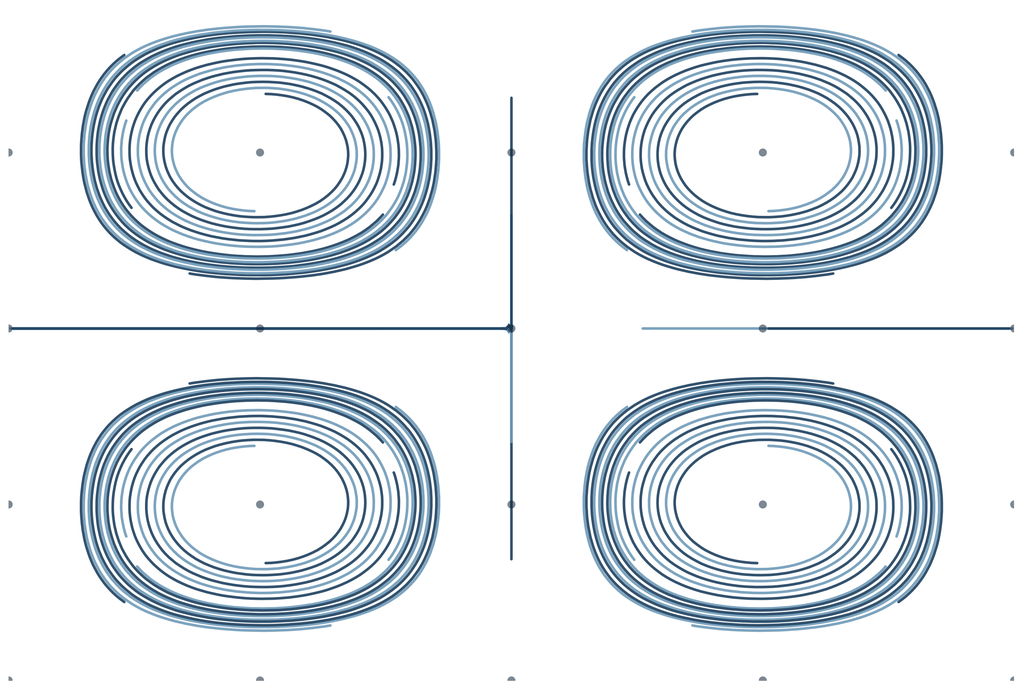
The idea

A survey of a geometric approach developed by the author over fifteen years. The behaviour of a viscous incompressible fluid becomes a property of curvature rather than a property of a vector field.

$$\mathrm{Ric}\,(g_{14}) = 0 \iff \text{Navier–Stokes}$$

the fourteen-dimensional metric is Ricci-flat exactly on the solutions of the system

The space splits into a flat six-dimensional part and two dual quadruples of coordinates, Eulerian and Lagrangian. The two classical descriptions of a fluid appear as two halves of one geometry.



A three-dimensional analogue of the Taylor–Green vortex built from Cartan invariants of the associated metric.

Six-dimensional companion

An associated six-dimensional metric is considered whose integrability condition is exactly the incompressibility of the fluid. Curvature invariants of this metric give coordinate-free characteristics of a flow.

Tools

Ricci-flat metrics · dual coordinates · Cartan invariants · projective connections

Applications

Taylor–Green vortex in three dimensions · equations of rigid body rotation

Cartan geometry of second-order ODEs

The construction is linked with the projective geometry of second-order ordinary differential equations in the sense of E. Cartan, which supplies the invariants used above.

Where the results come from

- **1974** · analytic solution of the two-dimensional Korteweg–de Vries equation
- **1994** · geometrical properties of multidimensional nonlinear differential equations
- **2001** · Riemannian and Einstein–Weyl geometry in the theory of ODEs
- **2017** · equations of incompressible liquids in Lagrange variables
- **2024–2026** · Ricci-flat spaces of the Navier–Stokes system, the present survey

Thirty-eight papers, one hundred and forty-five citations. All texts are free to download.

What is claimed and what is not

- No solution of the regularity problem is claimed.
- The correspondence is exact: Ricci-flatness is equivalent to the equations, not an approximation.
- The reformulation offers a different vantage point and a working set of invariants.

1974 — first results

2010–2025 — the geometric programme

2026 — survey

Abstract

We survey a geometric approach to the Navier–Stokes equations developed by the author over the past fifteen years. The central object is a Riemannian space of fourteen dimensions whose metric is Ricci-flat precisely on the solutions of the Navier–Stokes system, so that the behaviour of a viscous incompressible fluid becomes a property of curvature.

The space decomposes into a flat six-dimensional part and two dual quadruples of coordinates, Eulerian and Lagrangian, which exhibits the two classical descriptions of a fluid as two halves of a single geometry. We also recall the associated six-dimensional metric, whose integrability condition is the incompressibility of the fluid; the link with the projective geometry of second-order ordinary differential equations in the sense of E. Cartan; and the use of Cartan invariants to construct a three-dimensional analogue of the Taylor–Green vortex. An application of the same construction to the equations of rotation of a rigid body is indicated.

arXiv:2609.13314 · arxiv.org/abs/2609.13314

All papers, 1974–2026: **dryuma.com**